Combining Monte Carlo and worst-case methods for trajectory prediction in air traffic control: a case study

E. Crisostomi*, A. Lecchini-Visintini† and J. Maciejowski‡

* PhD. student, Department of Electrical Systems and Automation, University of Pisa, Italy

† Lecturer, Department of Engineering, University of Leicester, UK

‡ Professor, Department of Engineering, University of Cambridge, UK

Abstract — We illustrate, through a case study, a novel combination of probabilistic Monte Carlo methods and deterministic worst-case methods to perform model-based trajectory prediction in Air Traffic Control. The objective is that of computing and updating predictions of the trajectory of an aircraft on the basis of received observations. We assume that uncertainty in computing the predictions derives from observation errors, from the action of future winds and from inexact knowledge of the mass of the aircraft. Our novel approach provides worst-case prediction sets in which the future trajectory of the aircraft is guaranteed to belong and, at the same time, an empirical distribution of the most probable trajectories which can be used to compute various estimates such as the probability of conflict and the expected time of arrival. The case study is developed using the aircraft performance model developed by the EUROCONTROL Experimental Centre in BADA (Base of Aircraft DAta).

I. INTRODUCTION

The ability to compute a reliable prediction of the trajectory of an aircraft on a future horizon of the order of tens of minutes is an essential part of Air Traffic Control (ATC). Increasing levels of traffic both in Europe and in the US demand for more advanced trajectory prediction algorithms in order to sustain the performance of ATC - see e.g. Paglione et al. [1].

A trajectory prediction is calculated on the basis of the aircraft estimated position and state, some intent information, weather information and a performance model. The aircraft position and state can be estimated from radar measurements or can be broadcast by the aircraft itself, such as in the Mode-S [2] and ADS-B [3] systems. The intent information includes controller instructions and operational procedures (e.g. how a descent is executed). The weather information includes predicted winds and temperature profiles. The performance model describes the aircraft dynamic behavior and is essentially needed only to calculate trajectories which include a vertical displacement because commercial aircraft in level flight can be well modeled by simple kinematic models - see e.g. Paielli[4].

In a prediction on a future horizon of the order of tens of minutes there is unavoidable uncertainty. In the seminal papers of Paielli and Erzberger[5], [6], on the use of trajectory prediction to assess the probability of a future loss of safe separation between two aircraft (conflict probability), the approach to take into account uncertainty is to superimpose a distribution of position errors to a predicted nominal trajectory. The shape of the distribution of position errors is estimated on the basis of previous radar track records - see also Yang and Kuchar[7]. In Hu et al.[8] simple kinematic models driven by a stochastic wind field are used to investigate on the effect of spatial wind correlation on collision probability in level flight. Chaloulos and Lygeros[9] present a similar study based on Monte Carlo simulations of a more sophisticated wind model. The problem of estimating the probability of future conflicts and mid-air collisions has been an important benchmark for the development of advanced speed-up techniques for Monte Carlo methods – see e.g. Blom et al.[10]. Worst-case assumptions have been adopted for example in Tomlin et al.[11] for the purpose of designing safe maneuvers to resolve the encounter of a set of aircraft in level flight.

In this paper we present a case study devoted to the idea of combining Monte Carlo and worst-case methods to perform trajectory prediction. The idea of a combined worst-case and Monte Carlo methodology has been recently proposed by Balestrino et al.[12], [13]. In the case study we are concerned with the prediction of the trajectory of an aircraft on a leg of flight which includes a descent phase. The unknowns, in calculating the prediction from the point of view of ATC, are the mass of the aircraft and the action of the wind. These are realistic uncertainties in ATC[14], [15]. The purpose of the case study is to illustrate the advantages of using an aircraft performance model to calculate worst-case and probabilistic predictions at the same time. Here we adopt the aircraft performance model developed by the EUROCONTROL Experimental Centre in BADA (Base of Aircraft DAta)[16].

The paper is organized as follows. In the next section we review the Monte Carlo and worst-case approaches to estimation and prediction. In The case study we describe the design of the case study, i.e. the objective and the assumptions on the intent, the wind and the performance model. In Simulation example we illustrate the performance of our approach in different simulation scenarios. In the last section we state our findings and conclude the paper.

The solution of the case study has entailed the development of tailored algorithms to implement our approach within the full non-linear performance model developed in BADA [16]. A detailed technical presentation of the algorithms goes beyond the scope of this paper; a longer technical report is available upon request.

II. THE METHODS

In the stochastic Monte Carlo approach an empirical distribution of trajectory predictions is constructed by drawing random samples from Bayesian prior distributions on the initial state and on the unknowns (in our case the mass of the aircraft and the wind). The aim is to approximate the a posteriori distribution of the future trajectory given the priors on the unknowns and given the observations and the likelihood of observation errors. The approach extends the applicability of the popular Kalman filtering techniques to general non-Gaussian and non-linear models. On-line applications, such as trajectory prediction, require a computationally efficient implementation usually denoted sequential Monte Carlo or particle filtering - see e.g. Blom et al.[10], Van der Merwe et al.[17], Doucet et al.[18], Arulampalam et al.[19]. In particle filtering, the sampled predictions are computed sequentially on the basis of the last received observation without the need to reprocess older observations each time a new observation is received. The appeal of a Monte Carlo approach stems from the fact that it can be used in very complex problems and, in general, is straightforward to implement since it simply requires to run simulations of the model. The sampled trajectory predictions obtained in the Monte Carlo approach can be used to compute various estimates such as the probability of conflict with another aircraft and the expected time of arrival.

In the worst-case approach the aim is to compute guaranteed predictions in the form of sets containing all the trajectories which are consistent with the datum that the initial state, the unknowns and the observation errors belong to some given bounded uncertainty sets. In the context of estimation and filtering for dynamical systems this methodology is also referred to as the set-membership approach - see e.g. Bertsekas and Rhodes [20], Polyak et al.[21]. In the set-membership approach the prediction set is updated recursively. The initial predictions are only consistent with the given uncertainty sets of the initial state and of the unknowns; which are defined by existing knowledge on the model. In our case, the mass of the aircraft is known to be confined between a known minimum and a known maximum and it can be assumed that errors on the predicted winds respect to some credible bounds estimated from archived weather reports. The uncertainty sets are updated each time a new observation is received by excluding values of the unknowns which give rise to predictions which are not consistent with the received observation. Each time the uncertainty set of the unknowns is updated, a new set of guaranteed predictions is computed accordingly. The attractive feature of this approach is that the uncertainty set of the unknowns and the set of predictions are systematically reduced at the reception of each new observation while remaining guaranteed uncertainty sets in the worst-case sense.

The idea of combining Monte Carlo and set-membership methods has been proposed by Balestrino et al.[12], [13] as a novel solution to the problem of choosing representative values from the uncertainty set provided by a set-membership approach. The problem of choosing representative values is an important one because such values allow one to calculate useful quantities which can be used as 'indicative' estimates within the worst-case bounds. In the previous literature, this problem has been tackled with a deterministic approach consisting in the choice of nominal values corresponding to some geometrical definition of a center of the uncertainty set - see e.g. Bai et al.[22]. Balestrino et al.[12], [13] proposed instead the use of particle filters to construct an approximate a posteriori Bayesian distribution over the worst-case uncertainty sets. In this way one can use representative probabilistic estimates, such as the probability of conflict and the expected time of arrival, within the worst-case bounds. Balestrino et al.[12], [13] put forward this idea by developing combined Monte Carlo and set-membership algorithms for the case of a linear model. In this case study, we adopt the same general idea of combining Monte Carlo and set-membership methods but without restricting the scope to linear models because our aim is to employ the full non-linear performance model developed in BADA[16].

III. THE CASE STUDY

We consider a typical scenario in a Terminal Maneuvering Area (TMA) sector - see e.g. Lecchini-Visintini et al.[23]. In a TMA sector, aircraft, towards the end of their flight, descend from cruising altitude, around 30000 ft and above, to the entry points of the Approach Sector of the destination airport, which are typically between 5000 ft and 15000 ft. In our study, we specifically address the problem of performing trajectory prediction for an aircraft on a leg of flight composed by the following three phases: an initial phase in level flight at 30000 ft, followed by a descent to 10000ft and a final phase again in level flight at 10000ft.

The intent of the aircraft and the uncertainty affecting its trajectory from the point of view of ATC are illustrated in Figure 1. The intent is specified as follows. The aircraft is initially at 30000 ft in straight level flight. In the coordinate system of Figure 1, which has the horizontal axis aligned to the direction of the flight, the leg of flight of interest begins at (0 nmi, 30000 ft). The Top of Descent (ToD) is set at (10 nmi, 30000 ft). The aircraft will continue to travel in straight level flight and will start the descent phase when the ToD is reached. The descent phase will be executed at controlled vertical speed, or, equivalently, at controlled Rate of Climb OR Descent (ROCD). Here it is assumed that during the descent phase the pilot will use the Vertical Navigation

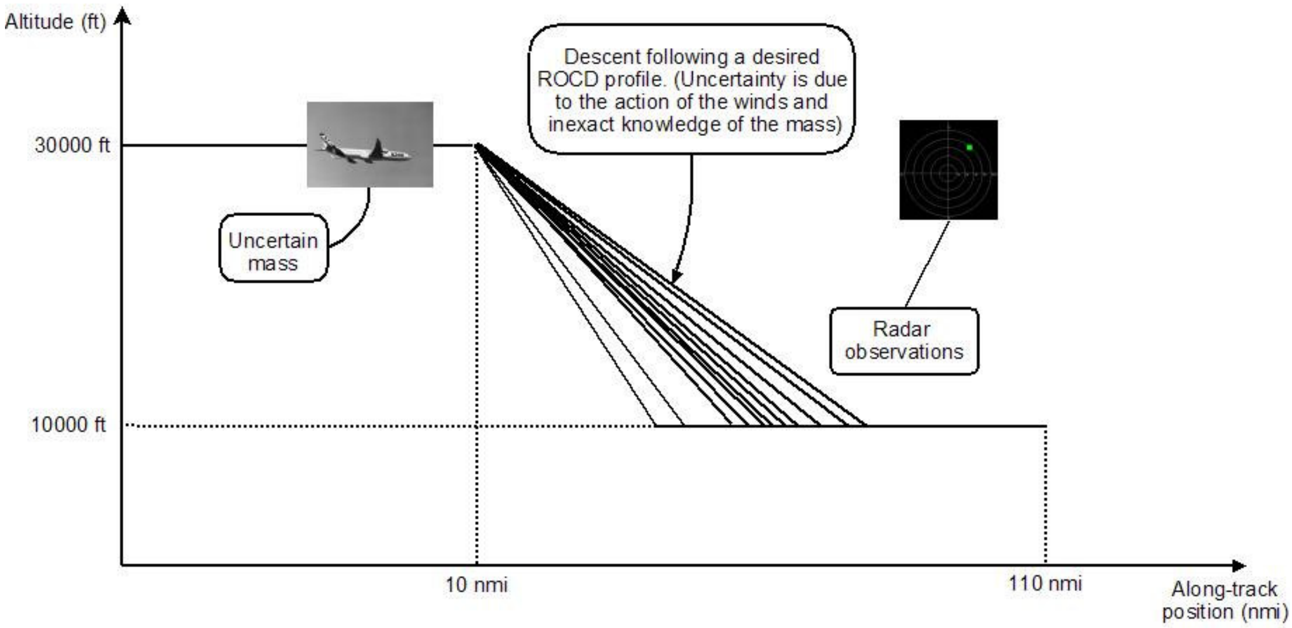

Fig. 1. A schematic representation of the flight considered in this case study.

(VNAV) system to follow a desired vertical path which has been issued by ATC. The aircraft will resume level flight when the altitude of 10000 ft is reached. The end of the leg of flight of interest is set at (110 nmi, 10000 ft).

We assume that the specified intent will be executed with no navigation errors. This assumption implies that: (i) the aircraft will fly on a straight route with null cross track error; (ii) the aircraft will begin the descent phase exactly when the ToD is reached; and (iii) the aircraft will execute the vertical path issued by ATC with no vertical navigation errors. From the point of view of ATC, uncertainty in the prediction of the aircraft trajectory will arise from the lack of knowledge of the exact mass of the aircraft in the prediction model, from the errors between the predicted and the actual winds encountered by the aircraft and from the observation errors. This uncertainty will affect mainly the prediction of the along track position of the aircraft, e.g. the location of the Bottom of Descent (BoD), and the Time of Arrival (TA). Our assumptions reflect the fact that current navigation systems allow the pilot to follow the ATC instructions with small errors and allow us to focus on the latter sources of uncertainty which would then be the predominant ones.

In the reminder of this section we introduce the models employed in this case study.

A. The observation model

We assume that radar observations are received every 6 sec and that the likelihood of observation errors has the form of a Gaussian density function with zero mean and variance σ^2= 500 m^2 truncated at 2σ. This assumption implies that each observation determines an along-track interval of length 4σ ≈ 90 m centered on the observation itself, in which the aircraft is guaranteed to be. Let us recall that the assumption that observation errors belong to a bounded set is required otherwise set-membership techniques cannot provide guaranteed predictions. Such an assumption corresponds to assume that outliers, i.e. completely wrong observations, never occur, or, if they occur, that they are detected and automatically discarded.

We assume that the aircraft airspeed is measured as well. This assumption is justified if Mode-S[2] or ADS-B[3] broadcast systems are in operation. In this case we assume that the likelihood of the air speed measurement errors has the form of a Gaussian density function with zero mean and variance σ^2=10 (m/s)^2 truncated at 2σ.

B. The wind model

In our case study the largest deviations from nominal predictions are caused by the action of the wind. Nominal wind predictions are usually available. However, an error in the prediction of the winds should still be taken into account in order to provide reliable trajectory predictions. A convenient choice is to consider the wind as having two components: a nominal one, which corresponds to the predicted wind, plus an additive component, which corresponds to the prediction error. We model the additive error as a zero mean random variable. Just for the sake of simplicity, we will assume that the nominal wind is zero. In our approach, a non-zero nominal component of the wind could be easily taken into account and be included in the model as a known offset to the mean of the 'stochastic wind'.

We assume that the wind remains constant at constant altitude. This assumption reflects the fact that winds at the same altitude are far more correlated than winds at different altitudes - see Cole et al. [24], Chaloulos and Lygeros [9]. In particular, this assumption can be expected to be realistic for the relatively small distances traveled in our case study. The model of the wind is based on an altitude grid consisting of 21 levels h_i^*, I = 1, .., 21 equally spaced at 1000 ft, from 10000 ft to 30000 ft. The values of the wind at these altitudes are generated as samples from a multivariate Gaussian distribution with correlation matrix which reproduce the vertical correlation of the wind. The support domain of the multivariate Gaussian distribution is truncated in such a way that the winds, and the difference between the winds at adjacent altitude levels, respect the following bounds:

$$\begin{cases} \left|w\left(h_i^*\right)\right| \le 20m/s & \forall i = 1,..,21 \\ \left|w\left(h_{i+1}^*\right) - w\left(h_i^*\right)\right| \le 20m/s & \forall i = 1,..,20 \end{cases} \tag{1}$$

(similar but less conservative bounds have been used in Kitsios and Lygeros [25]). In a similar way as it has been done before for the observation errors, these bounds are introduced in order to be able to calculate guaranteed predictions. It is important to introduce bounds also on the difference between the winds at adjacent altitude levels because the gradient of the wind has an important role in the aircraft performance model which will be introduced in the following subsection. The values of the winds at intermediate altitudes are generated by linear interpolation. A wind profile generated by our model is displayed in Figure 2. Notice that, since the wind velocity changes linearly between two adjacent levels, the gradient of the wind with respect to the altitude is piecewise constant.

The winds generated with our model are consistent with the accuracy studies performed on the database of the Rapid Update Cycle [24] performed by Schwartz et al. [26]. In their analysis the percentage of wind prediction errors greater than 10 m/s was 3 % overall, and 7 % in the worst month. Our model implies that the aircraft in level flight encounters a constant wind, and that the wind becomes instead a function of the altitude during the descent phase. More complex wind models can be easily introduced without affecting the applicability of our approach. Worst-case computations require only the values of the bounds on the winds introduced above. Monte Carlo methods require only to run many simulations of the adopted probabilistic wind model, such as the one used here, or a more complex one, such as the one used by Chaloulos and Lygeros[9].

C. The aircraft performance model

In level flight, aircraft maintain a constant airspeed which depends on the altitude. In this phase the motion of the aircraft is described well by a simple kinematic model - see e.g. Paielli [4]. Our assumption of null cross track errors simplifies the model to the following equation for the along track component:

$$x(k+1) = x(k) + v \cdot TS + w \cdot TS \tag{2}$$

where *TS* is the discretization step, *w* is the wind speed and *v* is the true airspeed of the aircraft. The true air speed in level flight is derived form the Calibrated Air Speed (CAS) which is a known constant parameter for each aircraft type. CAS is constant above 10000 ft until Mach transition altitude, and it can be converted into a True Air Speed (TAS) once the altitude level is known (see [16, eq (3.2-12)].). In eq [16, eq (3.2-12)] we

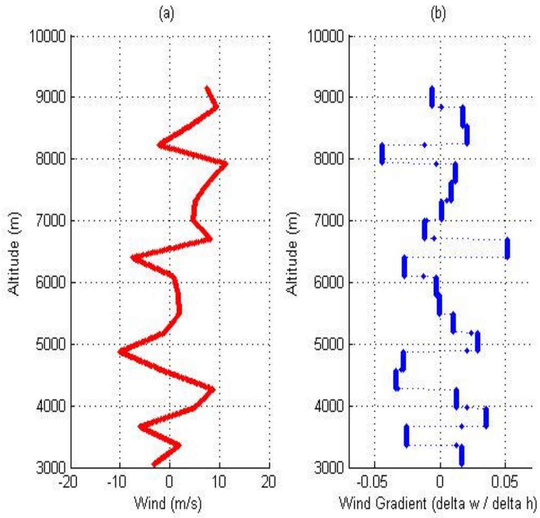

Fig. 2. (a) a possible realization of the wind profile; (b) the corresponding realization of the gradient of the wind with respect to altitude.

assumed that the pressure P_0, the temperature $Temp_0$ and the density ρ_0 at sea level are equal to their International Standard Atmosphere (ISA) values . The initial position $x(0)$ is supposed to be known within the accuracy of radar observations.

When the aircraft reaches the ToD, the model switches to a more complex one. In accordance with the BADA documentation [16], we can assume that during the descent the aircraft follows a nominal thrust profile which depends on the altitude. In addition, since we also assume that the aircraft follows a known vertical profile with no navigation errors, we actually fall under case (b) in [16, pag C7]. In this case, the ROCD, the altitude and the thrust in the BADA performance model become known at each step. Hence, the equations of the performance model can be written as:

$$\begin{cases} x(k+1) = & x(k) + v(k)\cos\gamma(k)TS + w(h(k))TS \\ h(k+1) = & h(k) + ROCD(k)TS \\ \gamma(k) = & \arcsin(ROCD(k)/v(k)) \\ v(k+1) = & v(k) + \dfrac{T(h(k)) - D(h(k),v(k),m(k))}{m(k)}TS \\ & - g \cdot \sin\gamma(k)TS - WG(h(k)) \cdot ROCD(k)\cos\gamma(k)TS \\ m(k+1) = & m(k) \end{cases} \tag{3}$$

where T is the thrust, D is the drag, g is acceleration gravity (9.81 m/s^2), γ is the flight path angle and WG is the gradient of the wind with respect to the altitude. The system of equation is written emphasizing the role of the ROCD.

The descent thrust T and the drag D in (3) are computed as in (4) and (6) using correction factors and nominal values that are typical of the particular aircraft and can be found in BADA [16]. From now on, for notational convenience, $T(h(k))$ and $D(h(k),v(k),m(k))$ will be simply denoted by $T(k)$ and $D(k)$.

We have:

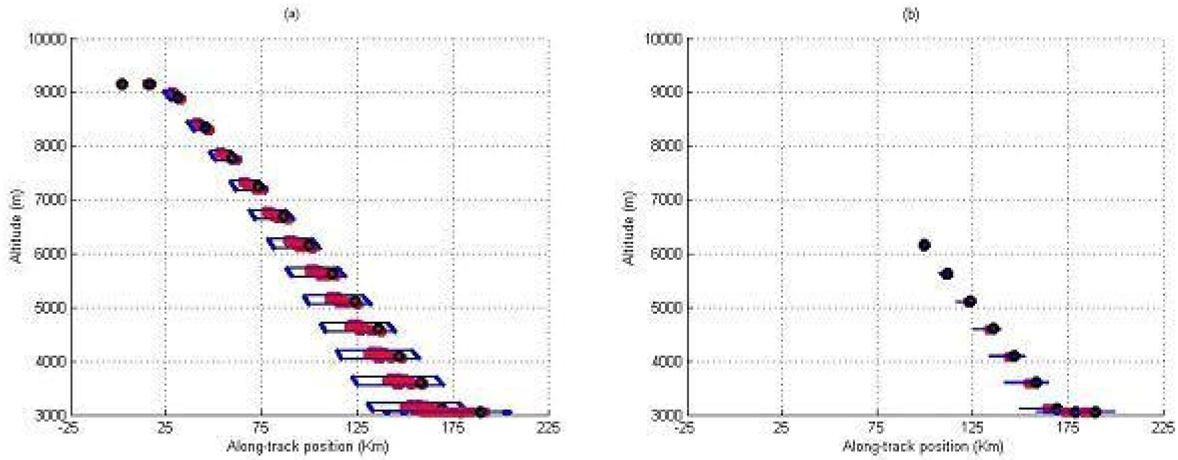

Fig. 3. Trajectory predictions drawn every 60 *sec*: (a) initial predictions; (b) half way predictions. The black circles represent the real trajectory; the red dots represent the most probable trajectories; the lines represent the guaranteed prediction

$$T(k) = C_{Tdes,high} \times T_{maxclimb}(k) \tag{4}$$

where $T_{maxclimb}(k)$ for a Jet engine type can be computed as

$$T_{maxclimb}(k) = C_{Tc1} \times \left(1 - \frac{h(k)}{C_{Tc2}} + C_{Tc3} \times h^2(k)\right) \tag{5}$$

and

$$D(k) = \frac{C_D(k) \cdot \rho(k) \cdot v^2(k) \cdot S}{2}, \tag{6}$$

where the drag coefficient $C_D(k)$ is computed as

$$C_D(k) = C_{D0,CR} + C_{D2,CR} \times (C_L(k))^2 \tag{7}$$

while the lift coefficient $C_L(k)$ is

$$C_L(k) = \frac{2 \cdot m(k) \cdot g}{\rho(k) \cdot v^2(k) \cdot S \cdot cos\,\varphi(k)} \tag{8}$$

In our case, the correction for the bank angle $\varphi(k)$, in the equation of the lift coefficient, can be neglected. In the drag equation (6), $\rho(k)$ is the air density, S is the wing reference area and $v(k)$ is the true airspeed as usual. Finally, $\rho(k)$ has been computed solely as a function of the altitude of the aircraft like:

$$\rho(k) = \rho_0 \left[\frac{Temp(k)}{Temp_0}\right]^{-\frac{g}{k_T R} - 1} \tag{9}$$

where R is the real gas constant for air, $R = 287.04 m^2 / (Ks^2)$, k_T is the International Standard Atmosphere (ISA) temperature gradient with altitude below the tropopause, $k_T = -0.0065 K/m$; ρ_0 and $Temp_0$ are density and temperature at sea level, here considered equal to their ISA values, $\rho_0 = \rho_{ISA} = 1.225 kg/m^3$, $Temp_0 = Temp_{ISA} = 288.15K$, and $Temp(k)$ can be computed as a function of the altitude $Temp(k) = Temp_0 - 6.5\frac{h(k)}{1000}$.

The unknown quantities in the above model are the mass *m*, the wind *w* and the gradient of the wind *WG*. We describe uncertainty on the mass through a uniform prior distribution between a maximum value and a minimum value which depend on the type of aircraft. The wind *w* and the gradient of the wind *WG* play the role of disturbances.

IV. SIMULATION EXAMPLE

In this section, we illustrate the performance of our combined worst case and Monte Carlo algorithms.

In the following simulations, the real trajectory of the aircraft is computed using randomly generated values of the unknowns and the coefficients of an A340 aircraft [16]. The actual initial position is −40 m. Each value of the unknowns has been generated according to its probabilistic model. The aircraft mass has been sampled between the minimum and the maximum values allowed for an A340 and is $2.3923 \cdot 10^5$Kg. The winds encountered during the flight are generated according to the wind model introduced in the previous section. The actual profile of the winds is the one shown in Figure 2. In the figures of this section the real position of the aircraft will be represented by black circles.

In Figure 3.(a) a trajectory prediction, made at the beginning of the flight, is displayed. In the figure, the predictions are displayed at intervals of 60 sec. The guaranteed prediction sets have a trapezoidal shape. The empirical distribution of trajectories generated by the Monte Carlo approach, which represents the most probable trajectories within the guaranteed set, is represented by red dots. Notice how the empirical distribution of the most probable trajectories is concentrated around the real trajectory and that the real

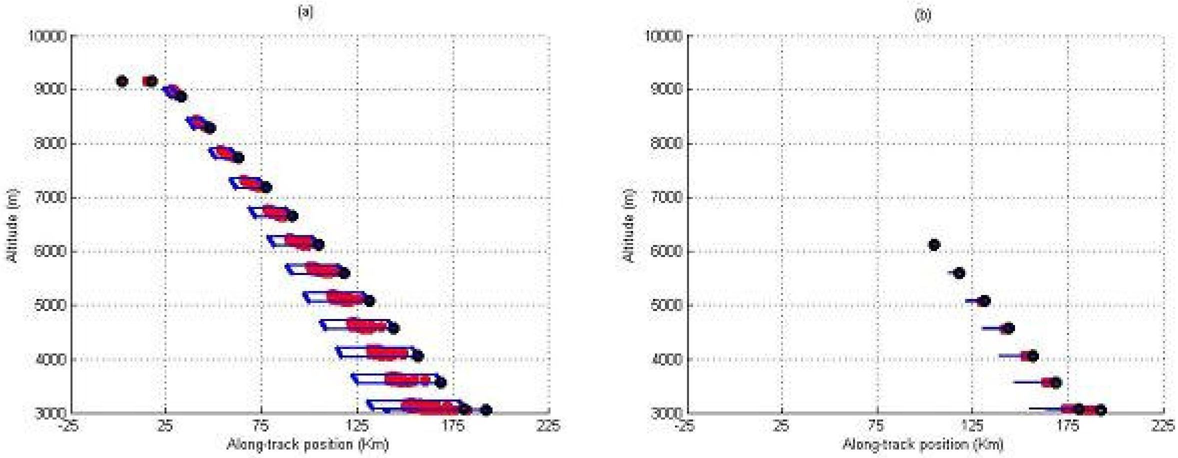

Fig. 4. Trajectory predictions in the actual worst case drawn every 60 *sec*: (a) initial predictions; (b) half way predictions. The black circles represent the real trajectory; the red dots represent the most probable trajectories; the lines represent the guaranteed prediction sets.

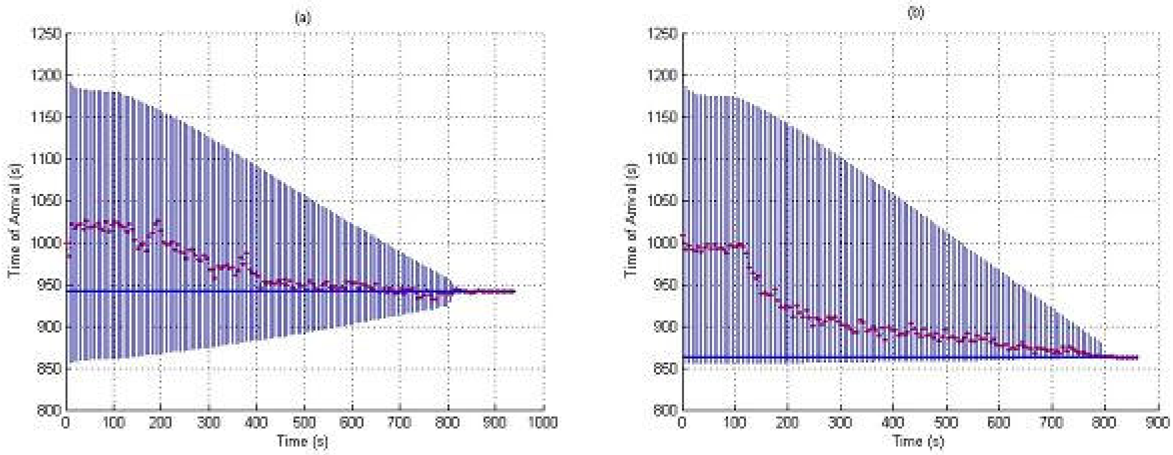

Fig. 5. Evolution of the predicted time of arrival during the flight: (a) in the case illustrated in Figure 3; (b) in the case illustrated in Figure 4. The lines represent the intervals in which the time of arrival is guaranteed to belong. The dots represent the expected time of arrival.

trajectory is always contained in the guaranteed prediction sets. In Figure 3.(b) a trajectory prediction made after the first half of the flight is displayed. In this figure, notice that the trapezoidal guaranteed predictions sets have collapsed to lines. The reason is that we assumed that the descent is executed with no vertical navigation errors. Hence, there is no uncertainty in the vertical displacement of the aircraft once the ToD has been passed. Figure 5.(a) displays the evolution of the guaranteed prediction intervals for the time of arrival, and of the expected time of arrival, during the flight.

An example where the aircraft is following a trajectory on the border of the feasible region is displayed in Figure 4. Figure 4.(a) displays again the trajectory prediction performed at the beginning of the flight, while Figure 4.(b) shows the prediction of the remaining trajectory when the aircraft has finished the first half of the flight. The reason why the aircraft is on the border of the guaranteed set is that, in this case, the mass of the aircraft and the wind profile were deliberately chosen to be at their worst admissible values. Notice that this situation is very unlikely to occur in practice, which is the reason why particles are far from the real trajectory in Figure 4.(a) and even after many measurements they still do not predict precisely the rest of the trajectory as shown in Figure 4.(b). Figure 5.(b) displays the evolution of the guaranteed prediction intervals for the time of arrival and of the expected time of arrival during this flight. Convergence to the real value is slower than in Figure 5.(a) because of the unlikely values of the uncertain quantities in this second case.

V. CONCLUSIONS

We have presented a case study devoted to the use of a combined worst-case and Monte Carlo method to perform trajectory predictions in Air Traffic Control. Each time a new observation becomes available, our algorithms provide:

- the worst-case prediction sets in which the aircraft trajectory is guaranteed to belong at each future time instant; and
- an empirical distribution which characterizes the most probable future trajectories and which can be used to compute estimates such as the expected time of arrival.

We envisage that our work will be useful to support novel conflict detection and resolution tools. An important aspect of our approach is that we have been able to employ the full non-linear aircraft performance model of BADA[16] without the need for the construction of any linearized approximate model, which is instead a common step in may other estimation and prediction methods. Future work will focus on the development of algorithms for the prediction of the trajectory during an 'open descent' when thrust and airspeed are controlled while ROCD is determined as consequence, i.e. case (a) in [16, pag C7]. In this case study we have assumed that the aircraft executes the prescribed intent with no navigation errors. Our approach does not impose any conceptual limitation to the development of prediction algorithms in which such an assumption is relaxed.

ACKNOWLEDGMENTS

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